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A PROJECT VARIABLE METRIC ALGORITHM FOR MATHEMATICAL PROGRAMS WITH EQUILIBRIUM CONSTRAINTS

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Abstract

By using a complementarity function, the mathematical program with equilibrium constraints (MPEC) problem is transformed into a nonlinear programming, and a new algorithm is proposed for the solution of (MPEC) problem. Under some suitable conditions, the proposed method is proved to possess not only global convergence, but also superlinear convergence rate.

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1. Introduction

We consider the following (MPEC) in which the constraints are defined by a nonlinear complementarity problem:

$$\min f(x, y) \quad \text{s.t.} \quad 0 \le F(x, y) \perp y \ge 0, \tag{1.1}$$

where $f : R^{n+m} \to R$, $F : R^{n+m} \to R^m$ are continuously differentiable, and $a \perp b$ denotes orthogonality of any vectors $a, b \in R^n$, i.e., $a^\top b = 0$. Such problems play an important role in many fields such as the design of transportation networks, economic equilibrium and engineering design etc., see [2].

Due to the existence of the complementarity constraints, it is a very difficult research to solve the (MPEC). There have been many scholars to study the (MPEC). Outrata et al. [3] pointed out that the Mangasarian-Fromovitz constraint qualication (MFCQ) does not hold at any feasible point of this (MPEC). So the theory for nonlinear programming can not be directly applied to the problem (1.1), hence standard methods are not guaranteed to solve such problems. [1] proposed a class of SQP algorithms for the solution of a kind of mathematical programs with linear complementarity constraints (MPLCC). However, they only possess global convergence.

In this paper, by means of the perturbed technique and a generalized complementarity function $\varphi(a, b, \mu) = a + b - \sqrt{a^2 + b^2 + 2\mu}$, $(a, b) \in R^2$, we transform equivalently problem (1.1) into a nonlinear equation constrained optimization problem. Then, we propose a new algorithm for the solution of (MPEC) problem by introducing a project variable metric algorithm. Under some suitable conditions, the proposed algorithm is proved to possess global convergence and superlinearly convergent rate.

2. Preliminaries, Equivalent Reformulations and Algorithm

The problem (1.1) is equivalent to the following reformulations:

$$\min f(x, y) \quad \text{s.t.} \quad w = F(x, y),\ w^\top y = 0. \tag{2.1}$$

The symbols we use in this paper are standard. For convenience, we list some of them as follows:

$$z = (x, y, w),\ t = (y, w),\ z^k = (x^k, y^k, w^k),\ t^k = (y^k, w^k),$$

$$dz = (dx, dy, dw),\ dt = (dy, dw),$$

$$dz^k = (dx^k, dy^k, dw^k),\ dt^k = (dy^k, dw^k),\ X_0 = \{z : w = F(x, y),$$

$$w^T y = 0\},$$

$$A^T = (a_1^T, a_2^T, \cdots, a_p^T),\ b^T = (b_1, b_2, \cdots, b_p),\ L = \{1, \cdots, m\}.$$

Throughout this paper, we suppose that the following assumptions hold:

H 2.1. The functions $f(x, y)$ and $F(x, y)$ are all twice continuously differentiable.

H 2.2. For all $z \in X_0$, the vectors $\{a_j(z) : j \in T\}$ are linearly independent.

Proposition 2.1. *Suppose that* $z^* \in X_0$ *satisfies the so-called nondegeneracy condition*:

$$(y_i^*, w_i^*) \neq (0, 0),\ i = 1, 2, \cdots m. \tag{2.2}$$

Then z^* *is a stationary point of the problem* (2.1) *if and only if there exist multipliers* $(\lambda^*, u^*) \in R^m \times R^m$ *such that*

$$\begin{pmatrix} \nabla f(x^*, y^*) \\ 0 \end{pmatrix} + \begin{pmatrix} \nabla F(x^*, y^*) \\ -I \end{pmatrix} \lambda^* + \begin{pmatrix} 0 \\ W^* \\ Y^* \end{pmatrix} u^* = 0, \tag{2.3}$$

where W^* *and* Y^* *are diagonal matrices with diagonal entries* w_j^* *and* y_j^*, $j = 1 \sim m$, *respectively*.

In this paper, we use a complementarity function $\varphi : R^2 \times [0, +\infty) \to R$ defined by

$$\varphi(a, b, \mu) = a + b - \sqrt{a^2 + b^2 + 2\mu}, \text{ for } (a, b, \mu) \in R^2 \times [0, +\infty). \tag{2.4}$$

For the complementarity function φ, we have the elementary property as follows:

$$\varphi(a, b, \mu) = 0 \Leftrightarrow a \geq 0, b \geq 0, ab = \mu. \tag{2.5}$$

This kind of function has wide applications to transform a complementarity constraint into an equality constraint. Obviously, $\varphi(\cdot, \cdot, \cdot)$ is differentiable at any $(a, b\ \mu) \neq (0, 0, 0)$,

$$\left(\frac{\partial \varphi(a, b\ \mu)}{\partial a}, \frac{\partial \varphi(a, b\ \mu)}{\partial b}\right)^T = \left(1 - \frac{a}{\sqrt{a^2 + b^2 + 2\mu}}, 1 - \frac{b}{\sqrt{a^2 + b^2 + 2\mu}}\right)^T, \tag{2.6}$$

and

$$0 < \frac{\partial \varphi(a, b, \mu)}{\partial a} < 2,\ 0 < \frac{\partial \varphi(a, b, \mu)}{\partial b} < 2,$$

$$\left(\frac{\partial \varphi(a, b, \mu)}{\partial a}\right)^2 + \left(\frac{\partial \varphi(a, b, \mu)}{\partial b}\right)^2 \geq 3 - 2\sqrt{2} > 0.$$

We now consider the following perturbed problem associated with problem (2.1):

$$\begin{aligned} & \min f(x, y) \\ (NLP_\mu) \quad & \text{s.t.} \quad w = F(x, y),\ \Phi(y, w, \mu) = 0, \end{aligned} \tag{2.7}$$

where $\Phi(y, w) : R^{2m} \to R^m$ is defined by

$$\Phi(y, w, \mu) = (\varphi(y_1, w_1, \mu), \cdots, \varphi(y_m, w_m, \mu))^T. \tag{2.8}$$

Let $z^k = (x^k, y^k, w^k) \in R^n \times R^m \times R^m$ and let $(y_i^k, w_i^k, \mu_k) \neq (0, 0, 0)$. The following equality holds:

$$\nabla_t \Phi(t^k, \mu_k) = (\Gamma_y^k, \Gamma_w^k)^T, \tag{2.9}$$

where

$$\Gamma_y^k = \Gamma(y^k, w^k, \mu_k) = \text{diag}(\gamma(y_i^k, w_i^k, \mu_k), i = 1 \sim m),$$

$$\Gamma_w^k = \Gamma(w^k, y^k, \mu_k) = \text{diag}(\gamma(w_i^k, y_i^k, \mu_k), i = 1 \sim m),$$

$$\gamma(a, b, \mu) = 1 - \frac{a}{\sqrt{a^2 + b^2 + 2\mu}}. \tag{2.10}$$

Obviously, it is easy to obtain the following result which indicates an equivalent relationship between (2.1) and (2.7).

Proposition 2.2. *Suppose* $z^* = (x^*, y^*, w^*)$ *and* (λ^*, u^*) *is the corresponding multipliers vector. Then* z^* *is a stationary point of problem* (2.1) *if and only if* $(z^*, \widetilde{v^*})$ *is a KKT pair of* (2.7) *for* $\mu = 0$ *with multipliers* $\widetilde{v^*} = (\lambda^*, \widetilde{v^*})$, *where*

$$\widetilde{u_i^*} = \begin{cases} w_i^* u_i^* & \text{if } i \in I_y(z^*) \overset{\Delta}{=} \{i : y_i^* = 0\}, \\ y_i^* u_i^* & \text{if } i \in I_w(z^*) \overset{\Delta}{=} \{i : w_i^* = 0\}. \end{cases} \tag{2.11}$$

For the sake of simplicity, we use the following notation:

$$H(z, \mu) = H(x, y, w, \mu) = (h_1(z, \mu), h_2(z, \mu), \cdots, h_{2m}(z, \mu))^T,$$

$$h_j(z, \mu) = \begin{cases} h_j(x, y, w, \mu) = F_j(x, y) - w_j & j = 1 \sim m, \\ h_j(x, y, w, \mu) = \varphi_j(y, w, \mu) & j = m + 1 \sim 2m, \end{cases}$$

$$g(z) = \begin{pmatrix} \nabla f(x, y) \\ 0 \end{pmatrix}, \nabla_{zz}^2 L(z) = \begin{pmatrix} \nabla_{xx}^2 f(x, y) & \nabla_{xy}^2 f(x, y) & 0_{n\times m} \\ \nabla_{yx}^2 f(x, y) & \nabla_{yy}^2 f(x, y) & 0_{m\times m} \\ 0_{m\times n} & 0_{m\times m} & 0_{m\times m} \end{pmatrix},$$

$$a_j(z) = \nabla h_j(z, \mu), j \in T \overset{\Delta}{=} \{1, 2, \cdots, 2m\}, A(z) = (a_j(z), j \in T),$$

$$J_+ = J_+(x, y, w, \mu) = \{i : h_i(z, \mu) > 0\},$$

$$J_0 = \{i : h_i(z, \mu) = 0\}, J_- = \{i : h_i(z, \mu) < 0\}.$$

We can rewrite the above problem (2.7) more compactly as

$$\min \quad f(x, y) \quad \text{s.t.} \quad H(z, \mu) = 0. \tag{2.12}$$

We define an auxiliary program problem (FP) of (2.12)

$$(FP) \quad \min G_c(z, \mu),$$

where c is a positive parameter and

$$G_c(z, \mu) = f(z) + c\|H(z, \mu)\|_1. \tag{2.13}$$

The directional derivative of $G_c(z, \mu)$ at the point z along the direction d is defined by

$$DG_c(z, \mu, d) = \lim_{\sigma \to 0+} \frac{G_c(z + \sigma d, \mu) - G_c(z, \mu)}{\sigma}.$$

The following result will be necessary in the next section.

Lemma 2.1 [5]. *$\forall z \in R^{n+2m}$, $d \in R^{n+2m}$, we have*

$$DG_c(z, \mu, d) = g(z)^T d + c \sum_{j \in J_+} a_j^T d + c \sum_{j \in J_o} |a_j^T d| - c \sum_{j \in J_-} a_j^T d.$$

Now, the algorithm for the solution of the problem (2.12) can be stated as follows.

Algorithm A:

Step 1. Given $z^1 \in R^{n+2m}$ and an initial symmetric positive definite matrix $B_1 \in R^{(n+2m)\times(n+2m)}$. Choose parameters $\tau \in (2, 3), \varepsilon > 0$, $\alpha \in (0, \frac{1}{2})$, $c_0 > 0$, $\mu_1 > 0$, $\hat{\varepsilon} > 0$. Set $k := 1$.

Step 2. For the current point z^k, compute

$$A_k = A(z^k),\ F_k = F(z^k) = (A_k^T B_k A_k)^{-1} A_k^T B_k,$$

$$P_k = P(z^k) = B_k(I_{n+2m} - A_k F_k),$$

$$d^k = -P_k g(z^k) - F_k^T H(z^k, \mu_k),$$

$$\widetilde{v^k} = -F_k g(z^k) + (A_k^T B_k A_k)^{-1} H(z^k, \mu_k). \tag{2.14}$$

Step 3. If $d^k = 0$, $\mu_k < \hat{\varepsilon}$, STOP. If $d^k = 0$, $\mu_k \geq \hat{\varepsilon}$, set $\mu_k = \frac{1}{2}\mu_k$, and go to Step 2. Otherwise, set $\mu_k = \frac{1}{2}\mu_k$, and go to Step 4.

Step 4. Compute $r_k = \max\ \{ | \widetilde{v_j^k}(z^k) | : j \in T,\ \text{and}\ h_j(z^k, \mu_k) \neq 0 \} + c_0$,

$$c_k = \begin{cases} \max\{ r_k,\ c_{k-1} + \varepsilon \} & c_{k-1} \le r_k, \\ c_{k-1} & c_{k-1} > r_k. \end{cases} \tag{2.15}$$

Step 5. Let

$$s^k = -F_k^T (\|d^k\|^\tau e + \widetilde{H}_k), \tag{2.16}$$

where $\widetilde{H}_k = (\widetilde{h_j^k}, j \in T) = (h_j(z^k + d^k, \mu_k), j \in T)$, $e = (1, 1, \cdots, 1)^T \in R^{|T|}$. If

$$G_{c_k}(z^k + d^k + s^k,\ \mu_k) \le G_{c_k}(z^k,\ \mu_k) + \alpha D G_{c_k}(z^k,\ \mu_k,\ d^k), \tag{2.17}$$

set $\lambda_k = 1$, $q^k = d^k + s^k$, go to Step 7.

Step 6. Compute λ_k, the first number λ in the sequence $\{1, \frac{1}{2}, \frac{1}{4}, \cdots\}$ satisfying

$$G_{c_k}(z^k + \lambda d^k,\ \mu_k) \le G_{c_k}(z^k,\ \mu_k) + \alpha\lambda D G_{c_k}(z^k,\ \mu_k,\ d^k), \tag{2.18}$$

set $q^k = d^k$.

Step 7. Set $z^{k+1} = z^k + \lambda_k q^k$, and compute a new symmetric positive define matrix B_{k+1} by means of some suitable technique. Let $k = k + 1$, and go back to Step 2.

3. Global Convergence of Algorithm

In this section, we analyze the global convergence of the algorithm. The following assumption is required in subsequent discussions.

H 3.1. The sequence $\{z^k\}$ generated by the algorithm is bounded, and the sequence $\{B_k\}$ of matrices is uniformly positive definite, i.e., there exist two positive constants a_1 and b_1 such that $a_1 \|y\|^2 \le y^T B_k y \le b_1 \|y\|^2$, for all k, $y \in R^{n+2m}$.

Suppose $z^k \to z^*, B_k \to B_*,\ k \in K$. It is obvious that

$$F_k \to F_*,\ P_k \to P_*,\ d^k \to d^*,\ \tilde{v}^k \to \tilde{v}^*,\ k \in K. \tag{3.1}$$

Lemma 3.1. *If* $\{z^k\}_{k\in K} \to z^*$ *holds, then there exists a positive integer* k_0 *such that* $c_k \equiv c_{k_0} \overset{\Delta}{=} c$, *for all* $k \geq k_0$.

According to Step 4, we know that $c_k \geq |\tilde{v}_j^k| + c_0,\ j \in T$. Therefore, from Lemma 3.1 and (3.1), it holds that

$$c \geq |\tilde{v}_j^*| + c_0 > |\tilde{v}_j^*|,\ j \in T. \tag{3.2}$$

Lemma 3.2. *If* $d^k = 0$ *or equivalently,* $DG_c(z^k, \mu_k, d^k) = 0$, *then* z^k *is a KKT point of (2.12). If* $d^k \neq 0$, *then* $DG_c(z^k, \mu_k, d^k) < 0$ *and* d^k *is a descent direction of (FP) at the point* z^k.

Proof. We divide the proof into two cases:

Case 1. Suppose $DG_c(z^k, \mu_k, d^k) = 0$. In view of (2.14), we obtain that

$$F_k A_k = (A_k^T B_k A_k)^{-1} A_k^T B_k A_k = I,\ P_k A_k = 0,\ P_k B_k^{-1} P_k = P_k.$$

Furthermore, we get that

$$\begin{aligned} A_k^T d^k &= A_k^T(-P_k g(z^k) - F_k^T H(z^k, \mu_k)) = -A_k^T P_k g(z^k) - A_k^T F_k^T H(z^k, \mu_k) \\ &= -A_k^T B_k g(z^k) + A_k^T B_k A_k F_k g(z^k) - (F_k A_k)^T H(z^k, \mu_k) \\ &= -H(z^k, \mu_k), \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} g(z^k)^T d^k &= -g(z^k)^T P_k g(z^k) + \widetilde{v^k}^T H(z^k, \mu_k) \\ &\quad - H(z^k, \mu_k)^T (A_k^T B_k A_k)^{-1} H(z^k, \mu_k). \end{aligned}$$

From Lemma 2.1, we have that

$$DG_c(z^k, \mu_k, d^k) = g(z^k)^T d^k + c\sum_{j\in J_+}(a_j^k)^T d^k - c\sum_{j\in J_-}(a_j^k)^T d^k$$

$$\le -g(z^k)^T P_k g(z^k) + \sum_{j\in J_+}(c - \tilde{v}_j^k)(a_j^k)^T d^k$$

$$- \sum_{j\in J_-}(c + \tilde{v}_j^k)(a_j^k)^T d^k. \quad (3.4)$$

So, from (3.2) and $P_k B_k^{-1} P_k = P_k$, it holds that $g(z^k) + A_k v^{\tilde{k}} = 0$, $H(z^k, \mu_k) = 0$, which shows that z^k is a KKT point of (2.12).

Case 2. Suppose $DG_c(z^k, \mu_k, d^k) \neq 0$. From (3.4), Lemma 3.1, for $k \geq k_0$ large enough, the fact that z^k is not a KKT point of (2.12) implies that $DG_c(z^k, \mu_k, d^k) < 0$.

Theorem 3.1. *The algorithm A generates an infinite sequence* $\{z^k\}$ *whose any accumulation point* z^* *is KKT point of* (2.12).

Proof. Suppose that $\{z^k\}_{k\in K} \to z^*$. Firstly, from the definition of $G_c(z^k, \mu_k)$, it is easy to show that

$$G_c(z^k, \mu_k) \to G_c(z^*, 0), \ k \to \infty. \quad (3.5)$$

We divide the rest of the proof into two cases by the structure of the algorithm:

Case 1. If there exist a subsequence $K_1 \subseteq K(|K_1| = \infty)$, such that $\forall k \in K_1$, $z^{k+1} = z^k + \lambda_k q^k$ are generated by Step 5 and Step 7, then, in view of (2.17), we get that

$$0 = \lim_{k\in K_1}(G_c(z^{k+1}, \mu_k) - G_c(z^k, \mu_k)) \le \lim_{k\in K_1} \alpha DG_c(z^k, \mu_k, d^k) \le 0.$$

So, we know $DG_c(z^*, 0, d^*) = 0$. From Lemma 3.2, it is obvious that z^* is a KKT point of (2.12).

Case 2. Without loss of generality, suppose $\forall k \in K_1$, $z^{k+1} = z^k + \lambda_k q^k$ are generated by Step 6 and Step 7. Suppose by contradiction that z^k is not a KKT point of (2.12). Similar to Theorem 3.7 in [6], we can get that z^k is a KKT point of (2.12), too.

4. Superlinear Convergence

In this section, we will analyze and verify the superlinear convergence. We first give the following assumption:

H 4.1. Suppose z^* is an accumulation point of sequence $\{z^k\}$ produced by algorithm *A*, $B_k \to B_*$, $k \to \infty$, and the second order sufficient condition holds under KKT point pair $(z^*, \widetilde{v^*})$.

According to [5, 4], similarly, we can obtain the following result.

Lemma 4.1. *Suppose that assumptions* H2.1 ~ H4.1 *hold, then we have that* $\|s^k\| = O(\|d^k\|^2)$, *and*

$$\lim_{k\to\infty} z^k = z^*,\ \lim_{k\to\infty} d^k = 0,\ \lim_{k\to\infty} \widetilde{v^k} = \widetilde{v^*}.$$

In order to obtain the superlinear convergence, the following assumption is necessary:

H 4.2. $\|\widetilde{P}_k(B_k^{-1} - \nabla_{zz}^2 \widetilde{L}(z^k, \widetilde{v^k}, \mu_k))d^k\| = o(\|d^k\|)$, *where*

$$\widetilde{P_k} = I_{n+2m} - A_k(A_k^T A_k)A_k^T,\ \nabla_{zz}^2 \widetilde{L}(z^k, \widetilde{v^k}, \mu_k)$$

$$= \nabla_{zz}^2 L(z^k) + \sum_{j\in T} \widetilde{v_j^k}\ \nabla^2 h_j(z^k, \mu_k).$$

Lemma 4.2. *Suppose that assumptions* H2.1 ~ H4.2 *hold, then* $\lambda_k \equiv 1$, $z^{k+1} = x^k + d^k + s^k$, *for all sufficiently large k.*

Proof. For all sufficiently large *k*, set

$$\theta_k \triangleq G_c(z^k + d^k + s^k, \mu_k) - G_c(z^k, \mu_k) - \alpha DG_c(z^k, \mu_k, d^k)$$

$$= g(z^k)^T(d^k + s^k) + \frac{1}{2}(d^k)^T \nabla_{zz}^2 L(z^k)d^k + o(\|d^k\|^2)$$

$$+ c\sum_{j\in T}(|h_j(z^k, \mu_k) + (a_j^k)^T d^k + (a_j^k)^T s^k + \frac{1}{2}(d^k)^T \nabla^2 h_j(z^k, \mu_k)d^k$$

$$+ o(\|d^k\|^2)| - |h_j(z^k, \mu_k)|)$$

$$- \alpha(g(z^k)^T d^k + c \sum_{i \in T,\ h_j(z^k, \mu_k) > 0} (a_j^k)^T d^k$$

$$- c \sum_{i \in T,\ h_j(z^k, \mu_k) < 0} (a_j^k)^T d^k).$$

In view of Lemma 4.1, we have that

$$A_k^T s^k = -\|d^k\|^\tau e - \widetilde{H^k},\ (a_j^k)^T s^k = -\|d^k\|^\tau - h_j(z^k + d^k, \mu_k),\ j \in T,$$

$$(a_j^k)^T s^k + \frac{1}{2}(d^k)^T \nabla^2 h_j(z^k, \mu_k) d^k = -\|d^k\|^\tau + o(\|d^k\|^2),\ j \in T,$$

$$g(z^k)^T d^k = -(d^k)^T B_k^{-1} d^k + \sum_{j \in T} \widetilde{v_j^k} h_j(z^k, \mu_k),$$

$$g(z^k)^T (d^k + s^k) = -(d^k)^T B_k^{-1} d^k - \sum_{j \in T} \widetilde{v_j^k}\ (a_j^k)^T (d^k + s^k) + o(\|d^k\|^2),$$

$$-\sum_{j \in T} \widetilde{v_j^k}\ (a_j^k)^T (d^k + s^k) = \sum_{j \in T} \widetilde{v_j^k} h_j(z^k, \mu_k) + \frac{1}{2}(d^k)^T (\sum_{j \in T} \widetilde{v_j^k} \nabla^2 h_j$$

$$(z^k, \mu_k)) d^k + o(\|d^k\|^2). \tag{4.1}$$

Hence

$$\theta_k = (\alpha - \frac{1}{2})(d^k)^T B_k^{-1} d^k + \frac{1}{2}(d^k)^T (\nabla_{zz}^2 \widetilde{L}(z^k, \widetilde{v^k}, \mu_k) - B_k^{-1}) d^k$$

$$- (1 - \alpha) \sum_{j \in T} (c - |\widetilde{v_j^k}|)\ |h_j(z^k, \mu_k)| + o(\|d^k\|^2).$$

Set $\widetilde{P}_* = I_{n+2m} - A_*(A_*^T A_*)^{-1} A_*^T,\ A_* = (\nabla h_j(z^*, 0),\ j \in T),$ we get that $\widetilde{P}_k \to \widetilde{P}_*$. Let

$$d^k = \widetilde{P}_* d^k + \rho,\ \rho = A_*(A_*^T A_*)^{-1} A_*^T d^k,$$

hence, we have that

$$\|\rho\| = O(\|d^k\|),\ \|\rho\| = o(\|d^k\|) + O(\|h(z^k, \mu_k)\|).$$

So by $\alpha \in (0, \frac{1}{2})$, $c \geq |v_j^{\tilde{k}}| + c_0$, $j \in T$, we obtain that

$$\theta_k \leq (\alpha - \frac{1}{2})a_1\|d^k\|^2 + o(\|d^k\|^2)$$

$$-(1-\alpha)\sum_{j \in T}(c - |v_j^{\tilde{k}}|)|h_j(z^k, \mu_k)| + o(\|h(z^k, \mu_k)\|).$$

Therefore, we have $\theta_k \leq 0$ for all sufficiently large k.

Theorem 4.1. *Suppose the assumptions* H2.1 ~ H4.2 *hold*, *if* $\mu_k = o(\|d^k\|)$, *then the sequence* $\{z^k\}$ *produced by Algorithm A converges superlinearly to a stationary point* z^* *of* (2.1), *i.e.*, $\|z^{k+1} - z^*\| = o(\|z^k - z^*\|)$.

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